CHAPTER 4

HF FILTERS

TYPES OF HF FILTERS

Among the multiple ways of making filters, only the following are useable for HF (3 MHz to 30 MHz) and VHF (30 MHz to 300 MHz):

1) LC filters;
2) Continuous time g_m-C filters;
3) Mechanical filters;
4) Ceramic filters;
5) Quartz filters;
6) Surface Acoustic Wave (SAW) filters.

With the exception of g_m-C filters, all these filters are realized with discrete components. The g_m-C filters are the only ones that can be integrated on a chip while maintaining good enough performance at high frequency (< 300 MHz).

ATTENUATION, PHASE, AND GROUP DELAY

A filter is a linear circuit that discriminates between different frequencies. The frequencies that are not affected by the filter make up the passband, while the attenuated frequencies make up the stopband. Since the filter is linear, the transfer function can be defined:

$$H(j\omega) \equiv \frac{output\ signal}{input\ signal} = H_0 \cdot \frac{\prod_{k=1}^{M} (s - z_k)}{\prod_{k=1}^{N} (s - p_k)} = |H(j\omega)| \cdot e^{-j\phi(\omega)} \qquad (4.1)$$

where z_k are the transmission zeros, p_k are the transmission poles and $\phi(\omega)$ is the phase shift. The input and output signals are generally voltages, but more rarely currents. It is common in filter theory to use attenuation, defined by:

$$A(j\omega) \equiv -20\log(|H(j\omega)|) \qquad (4.2)$$

which thus corresponds to the log of the inverse of the transfer function. The values ω_k for which $A(j\omega_k) = 0$ are the attenuation zeros and the frequencies ω_k for which $A(j\omega_k) \to \infty$ are the attenuation poles, which correspond to the transmission zeros situated on the imaginary axis.
In certain cases, the filter has a phase shift corresponding to a certain constant delay for the passband frequencies. We then specify the group delay by:

$$\tau(\omega) \equiv \frac{d\phi(\omega)}{d\omega} \qquad (4.3)$$

CLASSES OF TRANSFER FUNCTIONS

Filters can be classified according to their attenuation characteristics:

1) Low-Pass Filter = LPF;
2) High-Pass Filter = HPF;
3) Band-Pass Filter = BPF;
4) Band-Reject Filter = BRF.

Diagrams of these four types of attenuation characteristics are presented in Fig. 4-1.

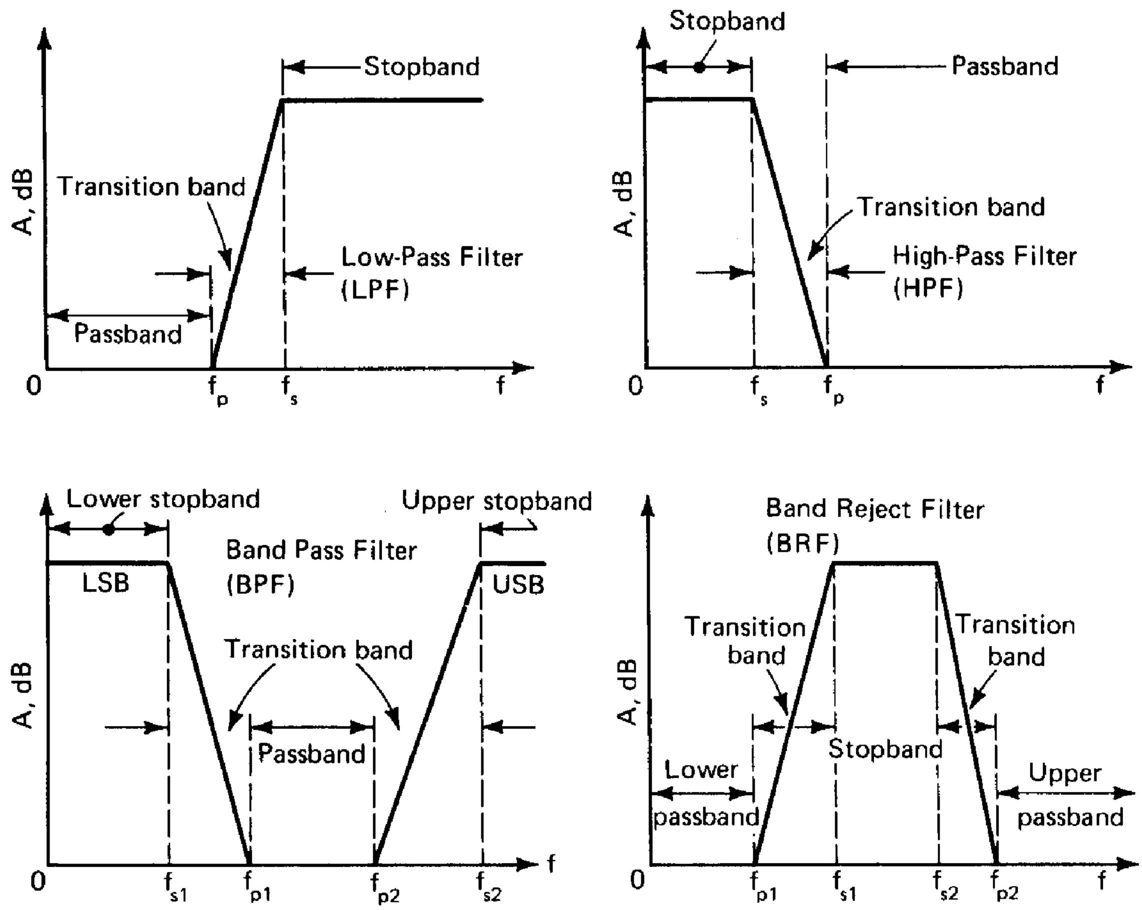
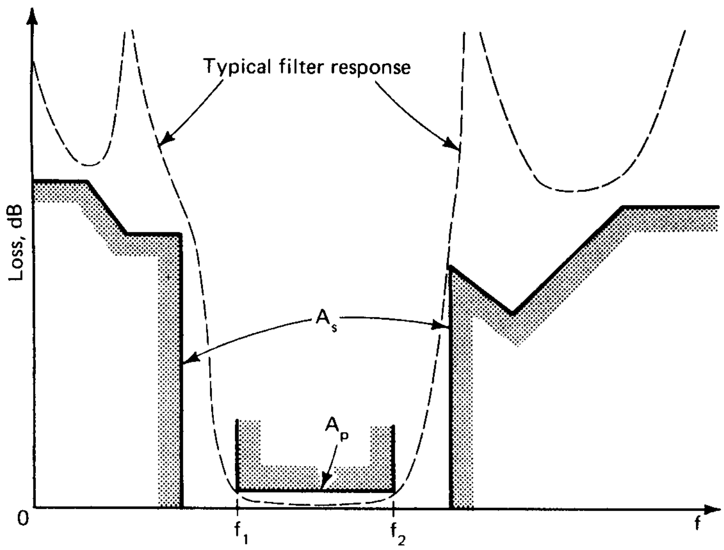


Fig 4-1: *The principal attenuation characteristics.*

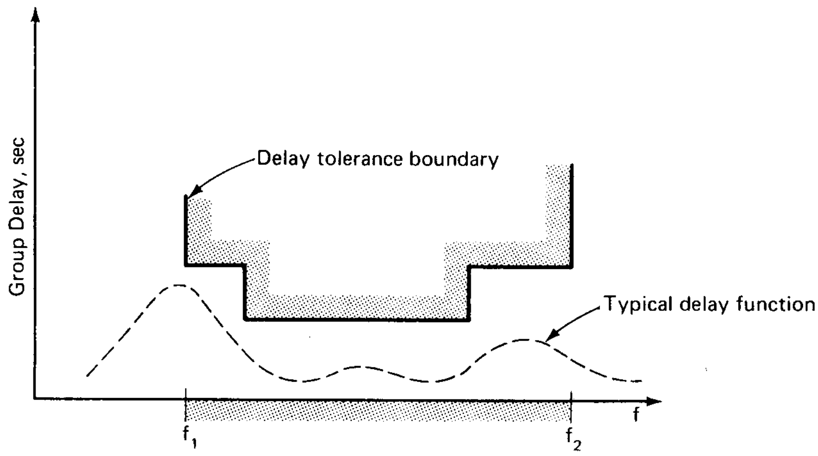
Each of these characteristics contains one or several passbands and stopbands, separated by transition bands in which the attenuation varies. The narrower these transition bands, the more selective the filter. This requires a high-order transfer function and implies a complex realization and high cost.

FILTER SPECIFICATIONS

Within the set of specifications of a filter, such as the dimensions, the power consumption, the functional temperature range, etc., the most important is certainly the specification of the attenuation characteristics or in certain cases the group delay. This is usually done given the attenuation or group delay tolerance limits (cf Fig. 4-2). It is useful to remember that the attenuation characteristics and the phase shift cannot be specified independently so that the causality of the filter is assured.



a) Attenuation vs. frequency specifications.



b) Group (phase) delay specifications.

Fig 4-2: *Different specifications of a filter.*

NORMALIZATION AND LOW-PASS PROTOTYPE

The design of a filter can be broken into two steps: the approximation step followed by the realization (or implementation). The approximation step involves the search for a transfer function that satisfies the imposed specifications, while the realization step involves the synthesis of a circuit (either active or passive) having the transfer function defined during the approximation step. In the case in which the specification is simple (constant and equal attenuation in the stopbands), tabulated analytical approximations can be used. These tables generally correspond to a low-pass filter with a stopband normalized to 1 rad/s (cf Fig. 4-3).

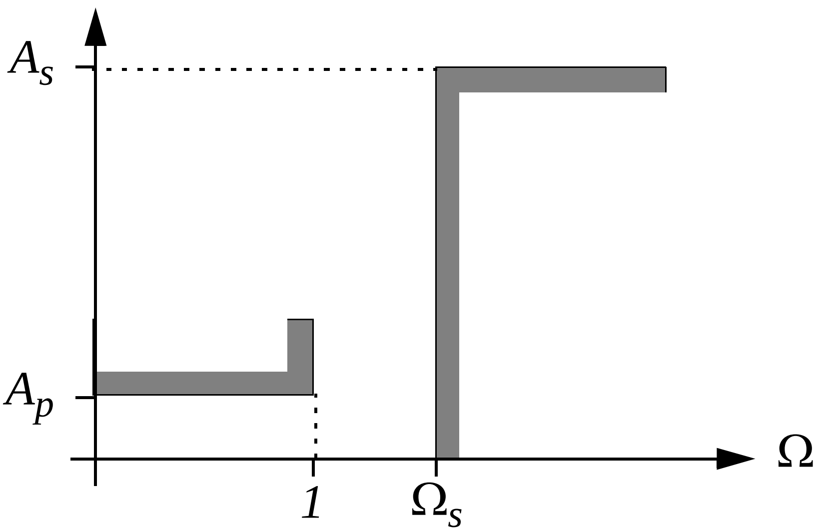


Fig 4-3: *Normalized tolerance plot.*

These tables can also be applied to high-pass, band-pass, or band-reject filter design, by using the appropriate frequency or circuit transformations.

LOW-PASS ↔ HIGH-PASS TRANSFORMATION

It is easy to transform a low-pass filter to high-pass by the simple inversion of the frequency axis, which corresponds to the following frequency transformation:

$$j\Omega = \omega_p / (j\omega) \tag{4.4}$$

where Ω is the normalized frequency corresponding to the low-pass prototype (LPP) filter and ω is the frequency of the high-pass (HP) filter to be realized, expressed in rad/s. The low-pass prototype filter is simply obtained by carrying out the transformations indicated in Fig. 4-4.

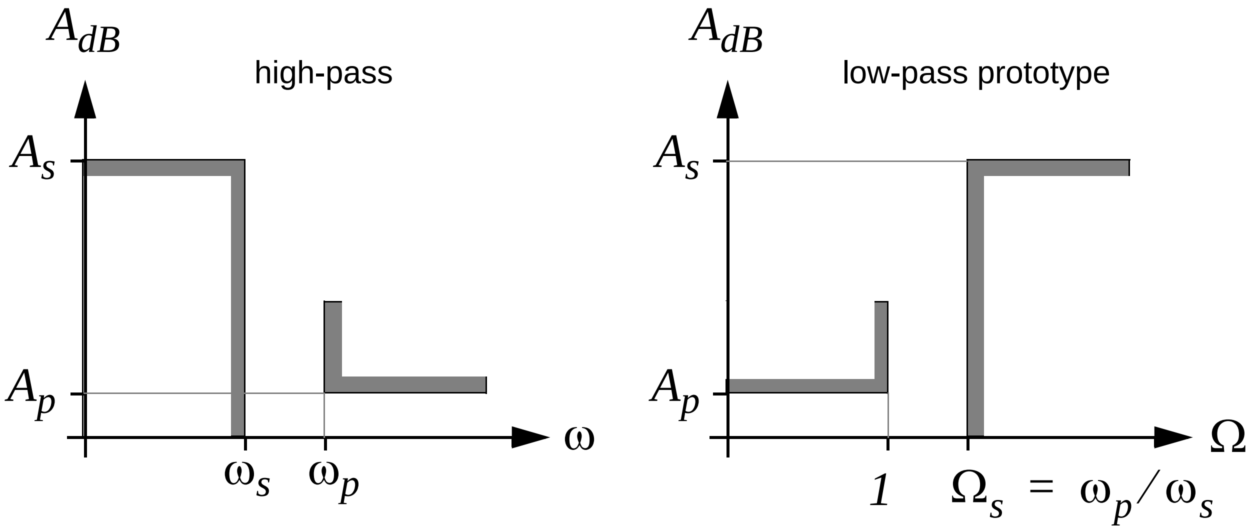
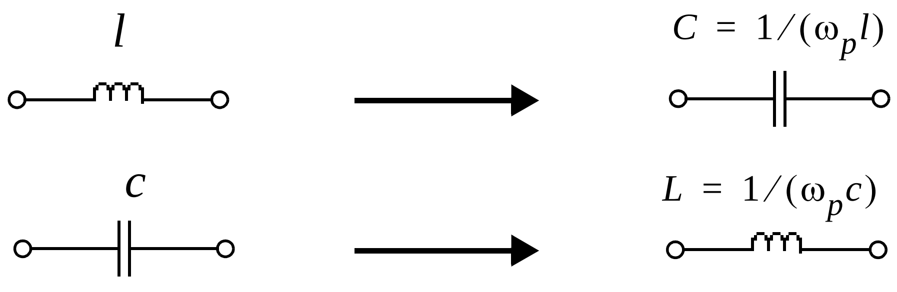


Fig 4-4: *Obtaining the LPP from the HP tolerance plot.*

Circuit transformations correspond to this frequency transformation. In particular, for LC filters, the (denormalized) high-pass filter is obtained by applying the transformation rules shown in Fig. 4-5.



a) Low-pass prototype filter. *b) Denormalized high-pass filter.*

Fig 4-5: *Transformation of elements of the LPP filter.*

LOW-PASS ↔ BANDPASS TRANSFORMATION (1/4)

In the case in which the tolerance plot of a bandpass filter specifies a constant and equal attenuation in the stopbands, the transformation low-pass ↔ bandpass can be used:

$$s = \frac{p^2 + \omega_0^2}{p \cdot B} \tag{4.5}$$

where $s = j\Omega$ is the Laplace variable in the domain of the low-pass prototype filter and $p = j\omega$ is the Laplace variable in the domain of the bandpass filter. Eqn. 4.5 transforms the frequency $s = 0$ to $p = \pm j\omega_0$, which is then the center frequency of the bandpass filter. The frequencies $s = \pm j\Omega$ are transformed to two positive frequencies $\omega_{1,2} = \mp \frac{1}{2}B\Omega + \sqrt{\omega_0^2 + \frac{1}{4}(B\Omega)^2}$ such that:

$$\omega_1 \omega_2 = \omega_0^2 \tag{4.6}$$

This signifies that each frequency of the tolerance plot of the low-pass prototype filter corresponding to a given attenuation is transformed to two frequencies having the same attenuation and located according to geometrical symmetry around the center frequency. In addition, the passband edges transform into two frequencies $\omega_{p1,p2} = \mp \frac{1}{2}B + \sqrt{\omega_0^2 + \frac{1}{4}B^2}$ and therefore:

$$\omega_{p2} - \omega_{p1} = B \tag{4.7}$$

where B is the filter bandwidth (in rad/s). The frequencies $s = \pm j\Omega_s$ map into two positive frequencies ω_{s1} and ω_{s2} such that:

$$\omega_{s2} - \omega_{s1} = B\Omega_s \tag{4.8}$$

These properties of the low-pass ↔ bandpass transformation are illustrated in an example in Fig. 4-6. The method to obtain the tolerance plot of the LPP from the bandpass filter specifications is described on page 4-10.

LOW-PASS $\leftrightarrow$ BANDPASS TRANSFORMATION (2/4)

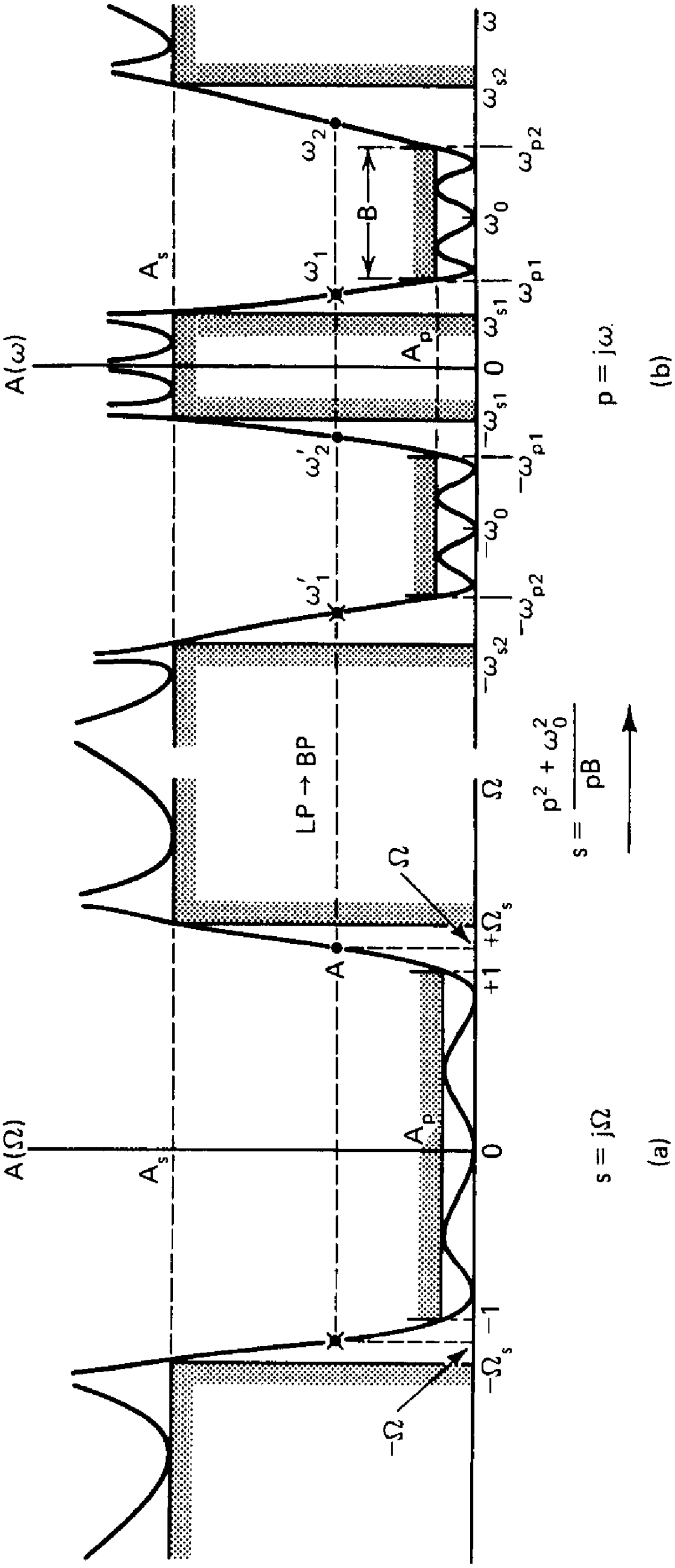


Fig 4-6: *Low-pass $\leftrightarrow$ bandpass transformation.*

LOW-PASS ↔ BANDPASS TRANSFORMATION (3/4)

Consider the tolerance plot of the bandpass filter shown in Fig. 4-7 a).

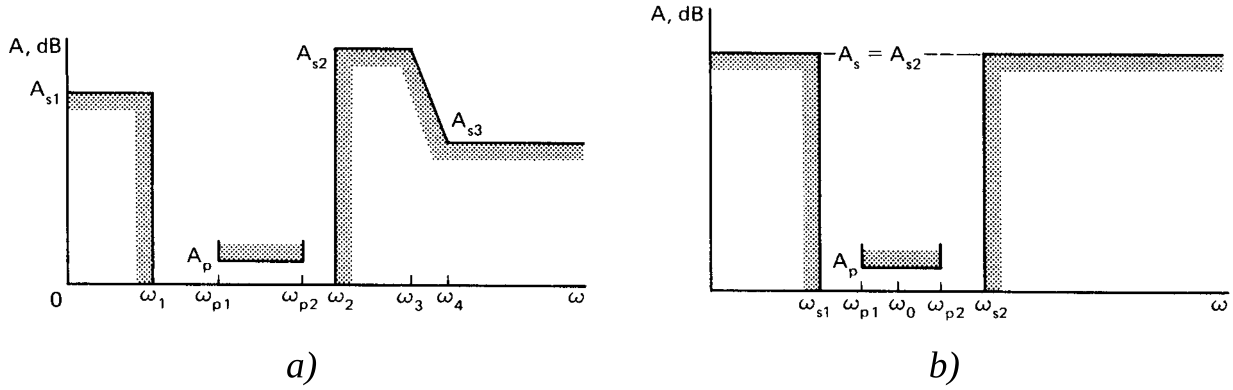


Fig 4-7: *Modification of the initial tolerance plot of the bandpass filter for the establishment of the specifications of the low-pass prototype filter.*

In order to be able to apply the low-pass ↔ bandpass transformation, the tolerance plot must be modified as follows:

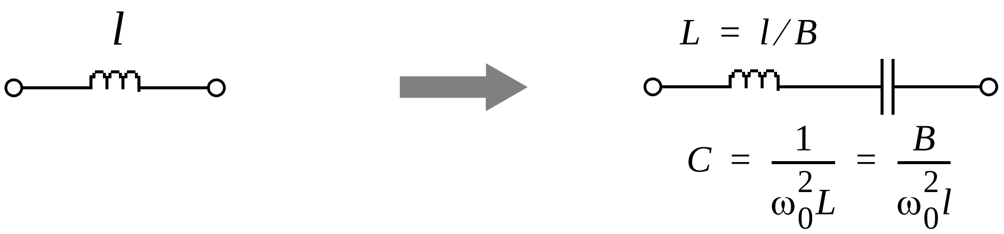
1) The attenuation in the stopbands must be leveled to the maximum attenuation (cf Fig. 4-7 b));
2) The stopband edges must be modified such that they satisfy the geometric symmetry property of the transformation. To do this, we first calculate the center frequency $\omega_0 = \sqrt{\omega_{p1}\omega_{p2}}$ and then evaluate $\omega_{s1} = \omega_0^2/\omega_2$. If $\omega_{s1} > \omega_1$, the stopband edges are ω_{s1} and ω_2. If $\omega_{s1} < \omega_1$, we must calculate $\omega_{s2} = \omega_0^2/\omega_1$ and define the stopband edges as ω_1 and ω_{s2}.

We can now derive the tolerance plot of the low-pass prototype (LPP) filter by remarking that the attenuations A_p and A_s remain unchanged, while the frequency Ω_s is given by:

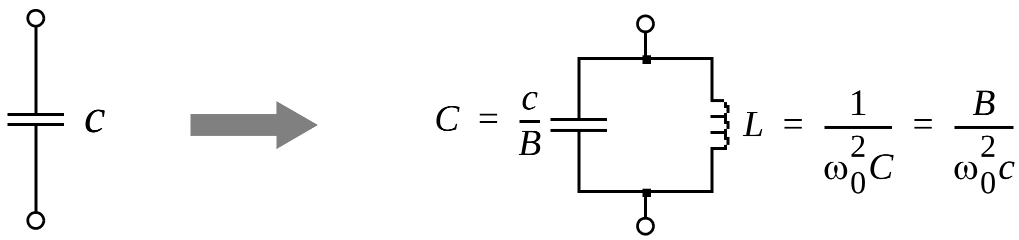
$$\Omega_s = \frac{\omega_{s2} - \omega_{s1}}{\omega_{p2} - \omega_{p1}} = \frac{\omega_{s2} - \omega_{s1}}{B} \tag{4.9}$$

LOW-PASS ↔ BANDPASS TRANSFORMATION (4/4)

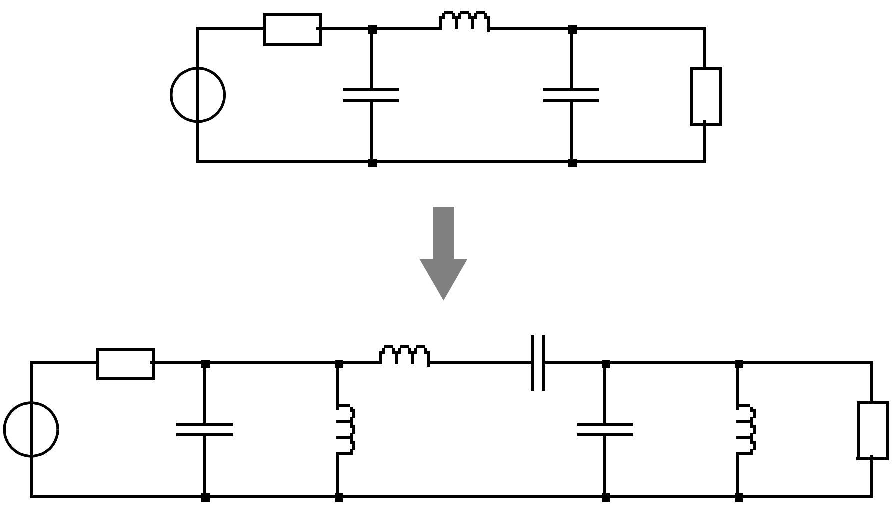
The low-pass ↔ bandpass transformation described by Eqn. 4.5 corresponds to a reactance transformation that can be directly applied to an LC filter. Knowing the center frequency ω_0 and the bandwidth B of the bandpass filter, one can then replace the inductors by series resonant circuits and the capacitors by parallel resonant circuits, all tuned to the same resonant frequency ω_0 expressed in rad/s.



a) Transformation of an inductor (normalized).



b) Transformation of a capacitor (normalized).



c) Transformation of a third-order all-pole filter.

Fig 4-8: *Application of the low-pass ↔ bandpass transformation.*

TYPES OF APPROXIMATIONS

There are several types of approximations, each having their own features. The best-known are:

1) *The Butterworth approximation:* offers a very flat attenuation in the passband with a monotonically increasing attenuation in the stopband. The transition from the passband to the stopband is controlled. The phase characteristic is nonlinear, and the group delay has a bump at the passband edge.
2) *The Chebyshev approximation:* offers a more rapid transition from the passband to the stopband than the Butterworth, but has ripples in the passband. The attenuation increases monotonically in the stopband. The phase characteristic is highly nonlinear and the group delay has peaks at the passband edge.
3) *The Bessel approximation:* offers a linear phase delay and therefore a constant group delay in the passband. However, the transition from the passband to the stopband is very gradual.
4) *The Cauer or Elliptic approximation:* offers a very steep transition from the passband to the stopband, but has ripples in the passband as well as the stopband. The phase characteristic and the group delay are highly nonlinear and rippled.

BUTTERWORTH APPROXIMATION (1/3)

The Butterworth function is certainly the simplest of the analytical approximations. The typical shape of the Butterworth transfer function is shown in Fig. 4-9.

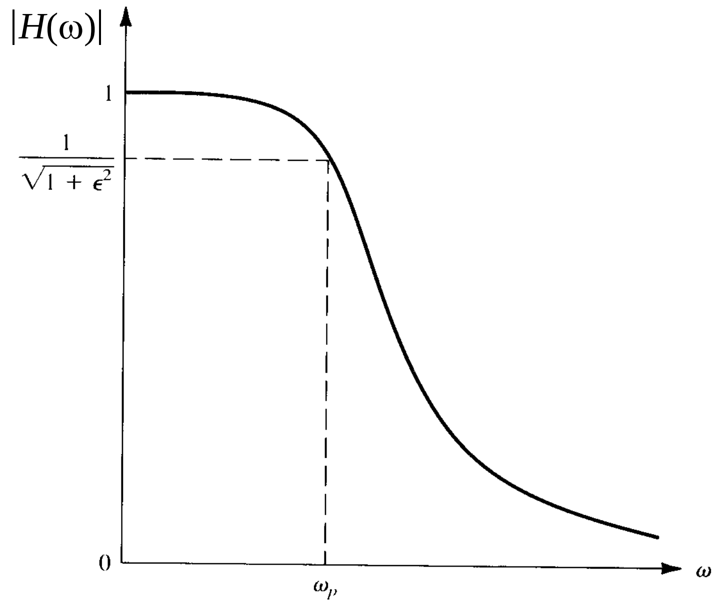


Fig 4-9: *Magnitude of the Butterworth transfer function.*

The magnitude of the Butterworth transfer function of order N and the corresponding attenuation are given respectively by:

$$|H(\omega)| \equiv \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} = \frac{1}{\sqrt{1+\varepsilon^2\Omega^{2N}}} \tag{4.10}$$

$$A_{dB} \equiv 20\log\left(\frac{1}{H(\Omega)}\right) = 10\log(1+\varepsilon^2\Omega^{2N}) \tag{4.11}$$

where $\Omega \equiv \omega / \omega_p$ is the frequency normalized to the cutoff frequency ω_p which is the frequency which corresponds to an attenuation A_p. In the particular case where $\varepsilon = 1$, ω_p corresponds to the frequency at -3 dB ($A_p = 3dB$). The parameter ε is thus determined by the maximum variation of the attenuation tolerated in the passband, given A_p:

$$\varepsilon = \sqrt{10^{A_p/10} - 1} \tag{4.12}$$

BUTTERWORTH APPROXIMATION (2/3)

The function given by Eqn. 4.10 is plotted for $\varepsilon = 1$ and for different values of N in Fig. 4-10.

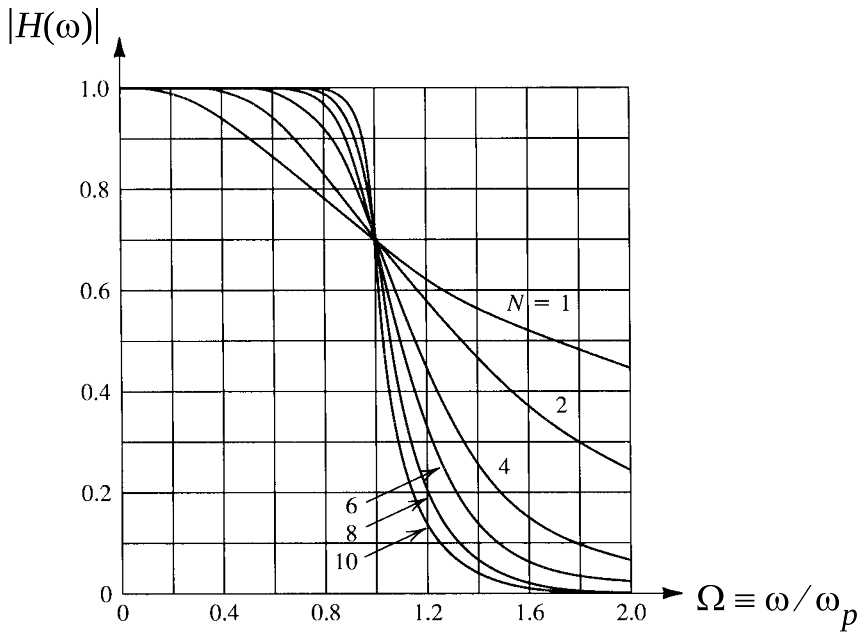


Fig 4-10: *Magnitude of the transfer function for* $\varepsilon = 1$.

It can be shown that the $2N–1$ derivatives of Eqn. 4.10 with ω cancel out at the origin, which explains the increasingly flat appearance of the characteristics near the origin when the filter's order increases. The order of the filter that satisfies a certain tolerance plot can be determined by noting that the attenuations at the passband and stopband edges are given by:

$$A_p = 10\log(1+\varepsilon^2) \qquad A_s = 10\log(1+\varepsilon^2\Omega_s{}^{2N}) \qquad (4.13)$$

from which one gets the minimum value of the order N that satisfies the tolerance plot:

$$N \geq \frac{\log\left(\dfrac{10^{A_s/10}-1}{10^{A_p/10}-1}\right)}{2\cdot\log(\Omega_s)} \qquad (4.14)$$

If the expression on the right side of Eqn. 4.14 is not a whole number, we must choose the next highest whole number.

BUTTERWORTH APPROXIMATION (3/3)

It can be shown that the poles of the transfer function $H(\Omega)$ are located on a circle as indicated in Fig. 4-11.

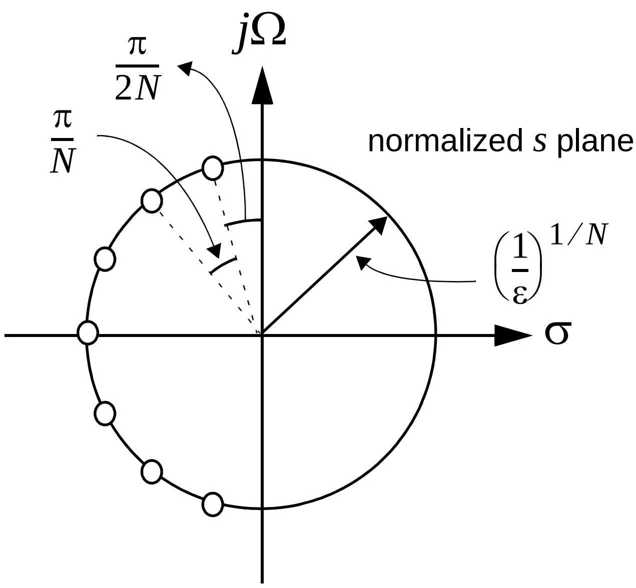


Fig 4-11: *Poles of the Butterworth transfer function in the normalized s plane (7th order).*

The complex conjugate poles can be grouped by pairs, and the transfer function can be factored into a product of a possible 1st degree function and 2nd order functions each having the same normalized resonant frequency Ω_0 and quality factors Q_k given by:

$$\Omega_0 \equiv \frac{\omega_0}{\omega_p} = \left(\frac{1}{\varepsilon}\right)^{1/N} \qquad Q_k = \frac{1}{2\sin\left(\frac{2k-1}{2N}\pi\right)} \qquad k = 1, 2, \ldots, N \tag{4.15}$$

The procedure for synthesizing a Butterworth filter is summarized below:

1) Normalize the tolerance plot by dividing the frequency by the filter's cutoff frequency. This permits Ω_s to be determined;
2) Determine the order according to Eqn. 4.14;
3) Calculate the value of ε using Eqn. 4.12.

CHEBYSHEV APPROXIMATION (1/3)

Fig. 4-12 shows the typical shape of the magnitude of the transfer function of even-order and odd-order Chebyshev filters. In contrast with the Butterworth filter, the Chebyshev filter has ripples in the passband, followed by a monotonic decrease in the stopband. Notice that the number of maxima and minima in the positive passband corresponds to $(N+1)$ where N is the order of the filter.

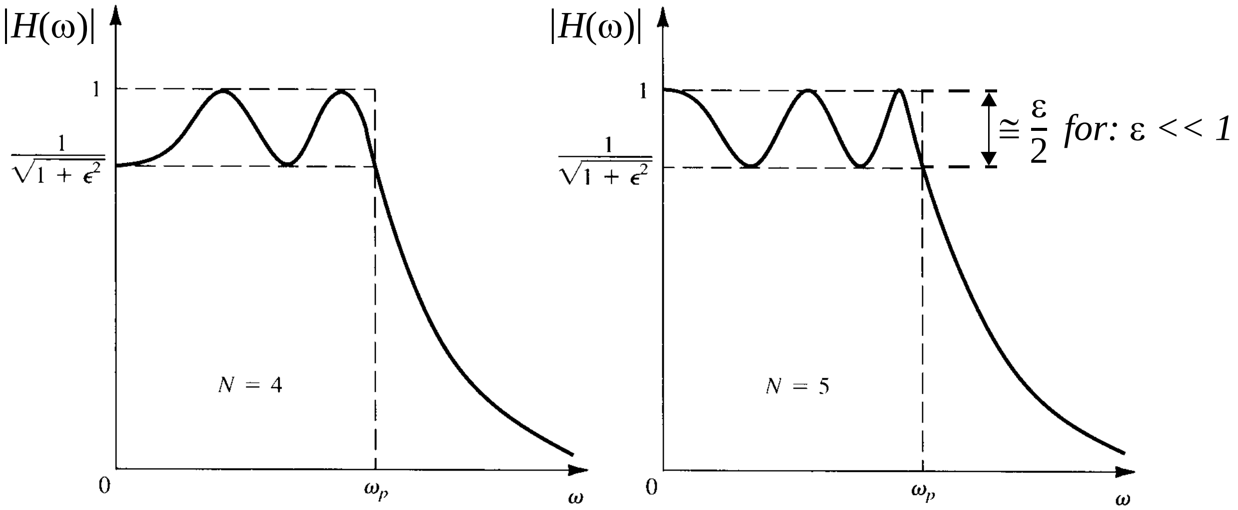


Fig 4-12: *Typical transfer function shapes for a Chebyshev filter.*

The transfer function is given by:

$$|H(\omega)| \equiv \frac{1}{\sqrt{1+\varepsilon^2 C_N^2\left(\frac{\omega}{\omega_p}\right)}} = \frac{1}{\sqrt{1+\varepsilon^2 C_N^2(\Omega)}} \tag{4.16}$$

$$A_{dB} \equiv 20\log\left(\frac{1}{|H(\Omega)|}\right) = 10\log(1+\varepsilon^2 C_N^2(\Omega)) \tag{4.17}$$

where $C_N^2(\Omega)$ is the Chebyshev polynomial of order N defined by:

$$C_N(\Omega) \equiv \begin{cases} \cos(N \cdot \mathrm{acos}(\Omega)) & \Omega \le 1 \\ \cosh(N \cdot \mathrm{acosh}(\Omega)) & \Omega > 1 \end{cases} \tag{4.18}$$

CHEBYSHEV APPROXIMATION (2/3)

These polynomials satisfy the following recursive formula:

$$C_{n+1}(\Omega) = 2\Omega C_N(\Omega) - C_{N-1}(\Omega) \tag{4.19}$$

The first polynomials given below are shown in Fig. 4-13.

$$C_0(\Omega) = 1 \qquad C_1(\Omega) = \Omega$$
$$C_2(\Omega) = 2\Omega^2 - 1 \qquad C_3(\Omega) = 4\Omega^3 - 3\Omega \tag{4.20}$$
$$C_4(\Omega) = 8\Omega^4 - 8\Omega^2 + 1$$

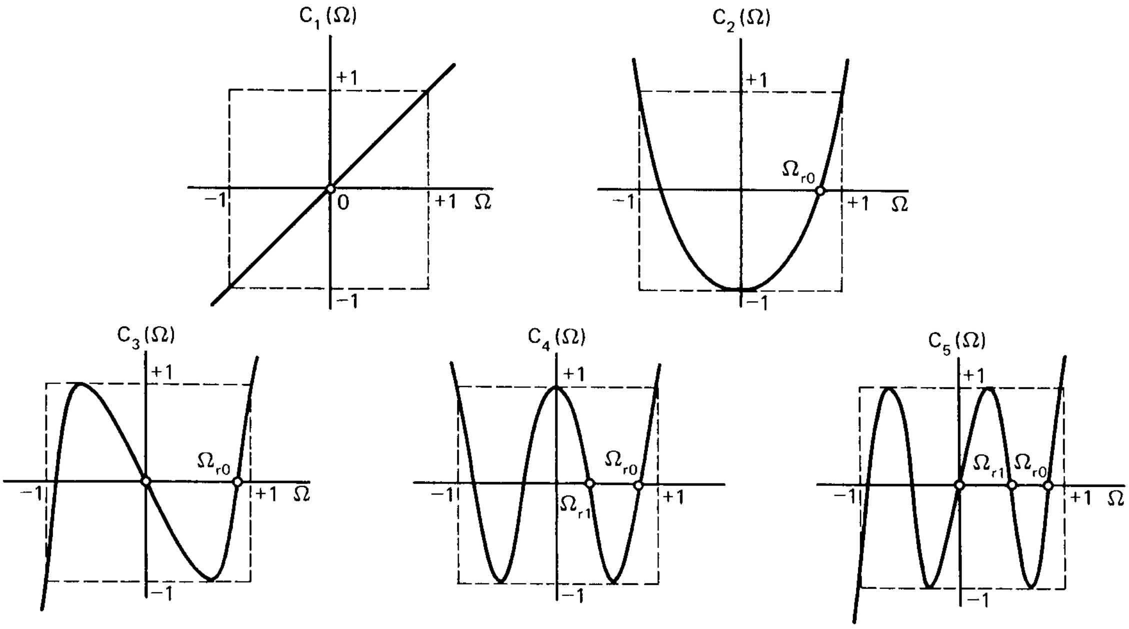


Fig 4-13: *Chebyshev polynomials.*

The attenuation of a Chebyshev filter of order N at the stop-band edge is given by:

$$A_s \equiv A_{dB}(\Omega = \Omega_s) = 10\log(1 + \varepsilon^2 C_N^2(\Omega_s)) \cong 20\log(\varepsilon C_N(\Omega_s)) \tag{4.21}$$

Which gives: $$A_s + 20\log\left(\frac{1}{\varepsilon}\right) = 20\log(C_N(\Omega_s)) \tag{4.22}$$

CHEBYSHEV APPROXIMATION (3/3)

The right side of Eqn. 4.22 is constant for a given order N and value Ω_s. Once the order has been chosen, the attenuation can be split up between the two left-hand terms of Eqn. 4.22 according to the specifications. An increase of the attenuation in the stopband implies an increase of ε and therefore of the ripple in the passband. Eqn. 4.22 is represented in Fig. 4-14. Knowing A_p , A_s and Ω_s , the chart in Fig. 4-14 can be used to determine the order necessary to satisfy the tolerance plot.

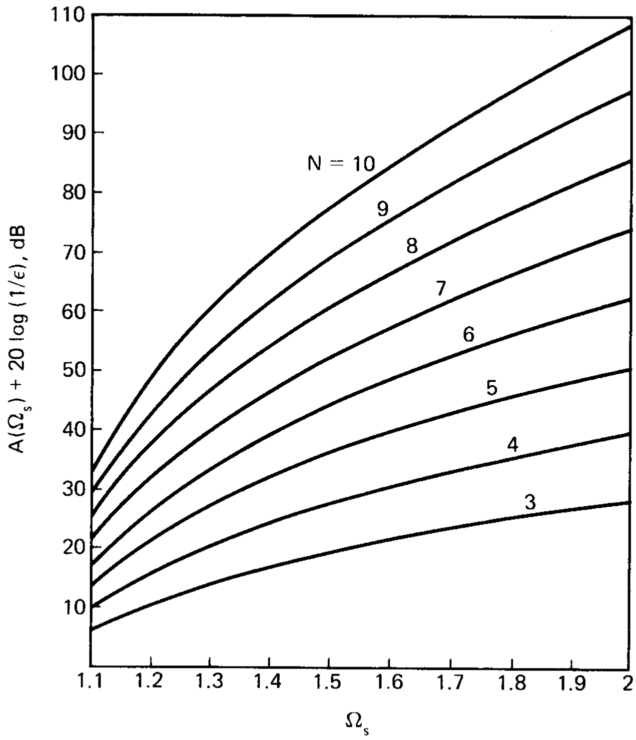


Fig 4-14: *Chart for choosing the Chebyshev filter order.*

The order necessary to satisfy the tolerance plot can also be determined by using the following formula:

$$N \geq \frac{A_s + 20\log\left(\frac{1}{\varepsilon}\right) + 6}{8.68\,\mathrm{acosh}(\Omega_s)} \qquad (4.23)$$

COMPARISON OF BUTTERWORTH AND CHEBYSHEV APPROXIMATIONS

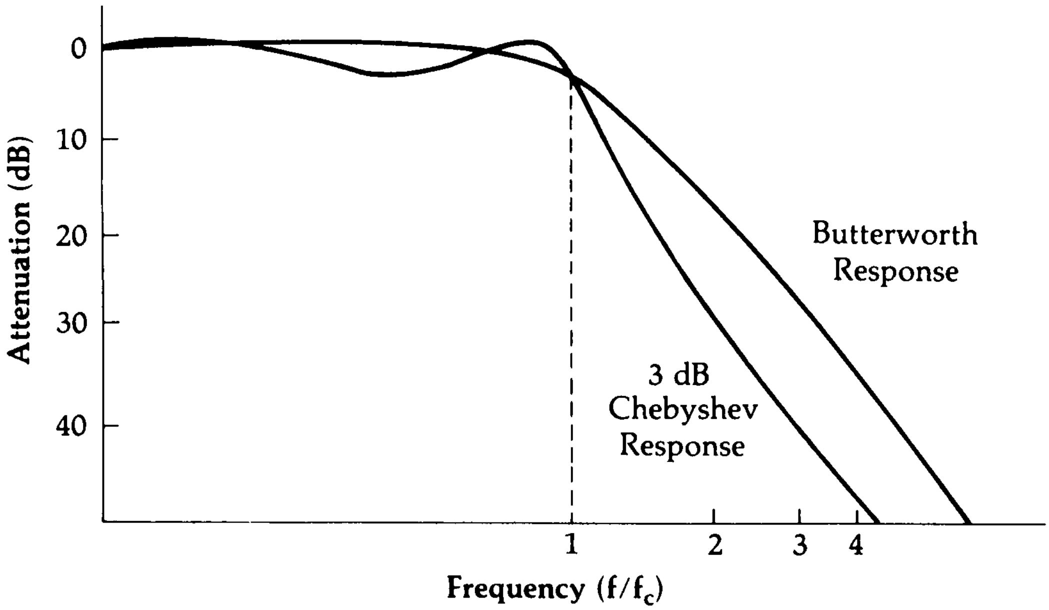
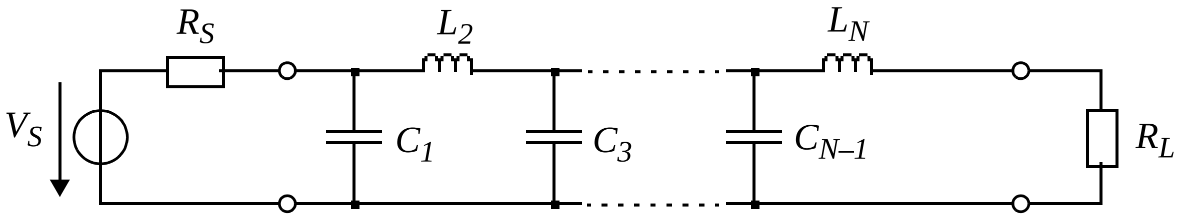


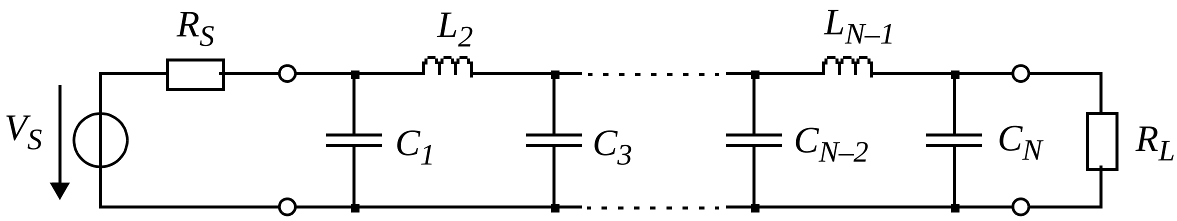
Fig 4-15: *Comparison of the attenuation of 3rd order Butterworth and Chebyshev filters.*

ALL-POLE LOW-PASS LC FILTER (1/3)

For both Butterworth and Chebyshev approximations, there are direct relationships between the values of the reactive components and the characteristic parameters ε and N for the low-pass prototype filters in Fig. 4-16 and 4-17. In general, we would choose one of the filters with the minimum number of inductors in Fig. 4-16.

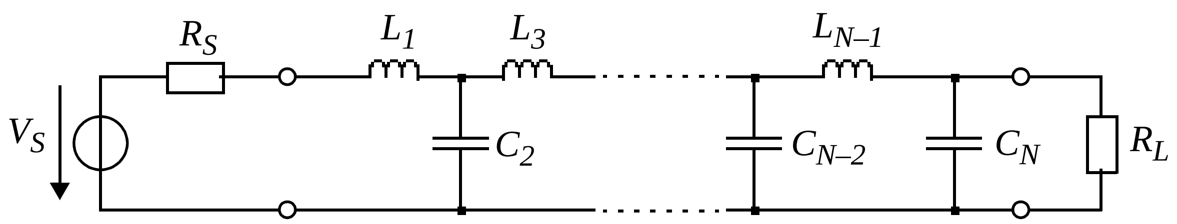


a) Even-order (N even).

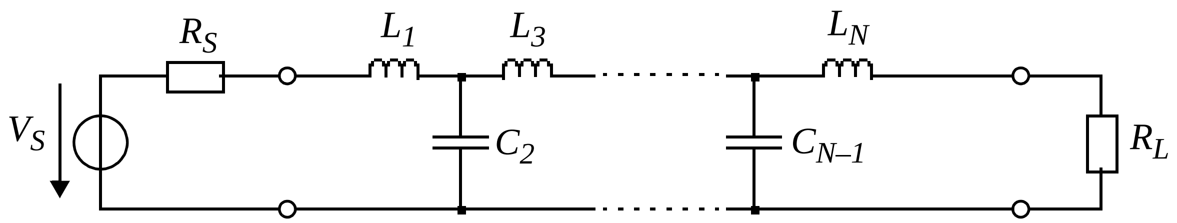


b) Odd-order (N odd).

Fig 4-16: *All-pole low-pass LC filters with minimum inductors.*



a) Even-order (N even).



b) Odd-order (N odd).

Fig 4-17: *All-pole low-pass LC filters with minimum capacitors.*

ALL-POLE LOW-PASS LC FILTER (2/3)

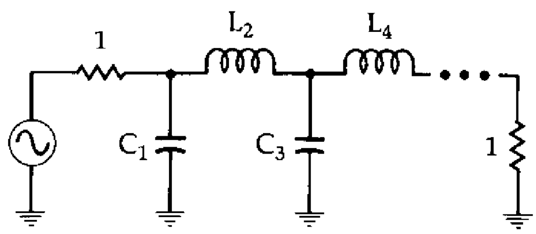
Butterworth

For the Butterworth approximation, the values of the reactive elements (capacitors et inductors) of the filters in Fig. 4-16 having *equal resistive terminations* equal to 1 Ω and a cutoff frequency ω_p equal to 1 rad/s, are simply given by:

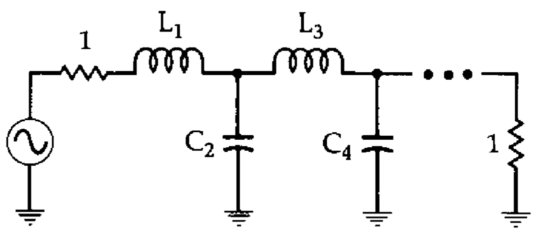
$$\boxed{C_k, L_k = 2\varepsilon^{1/N}\sin\left(\frac{2k-1}{2N}\pi\right) \qquad k = 1, 2, \ldots, N} \qquad (4.24)$$

Note that Eqn. 4.24 is only valid for filters with *identical terminations* $R_S = R_L = 1\Omega$. The values calculated from Eqn. 4.24 are tabulated in Table 4-1 for $\varepsilon = 1$. For the case in which $R_S \neq R_L$, it is necessary to use Table 4-2.

Table 4-1: *Butterworth prototype filter ($R_S = R_L = 1\Omega$) and ω_p = 1 rad/s.*



n	C_1	L_2	C_3	L_4	C_5	L_6	C_7
2	1.414	1.414					
3	1.000	2.000	1.000				
4	0.765	1.848	1.848	0.765			
5	0.618	1.618	2.000	1.618	0.618		
6	0.518	1.414	1.932	1.932	1.414	0.518	
7	0.445	1.247	1.802	2.000	1.802	1.247	0.445
n	L_1	C_2	L_3	C_4	L_5	C_6	L_7



ALL-POLE LOW-PASS LC FILTER (3/3)

Chebyshev

The relationships for a Chebyshev approximation are a little more complicated. First we must determine the constants h and ξ using the value of ε:

$$h \equiv \left(\frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}}\right)^{1/N} \qquad \textit{and:} \qquad \xi \equiv h - \frac{1}{h} \tag{4.25}$$

The cutoff frequency ω_p is equal to 1 rad/s. The values of the reactive elements are then given by:

$$C_1 = \frac{4\sin\left(\frac{\pi}{2N}\right)}{\xi R_S} \tag{4.26}$$

$$C_{2k-1} \cdot L_{2k} = \frac{16\sin\left(\frac{4k-3}{2N}\pi\right)\sin\left(\frac{4k-1}{2N}\pi\right)}{\xi^2 + \left(2\sin\left(\frac{2k-1}{N}\pi\right)\right)^2}$$

$$C_{2k+1} \cdot L_{2k} = \frac{16\sin\left(\frac{4k-1}{2N}\pi\right)\sin\left(\frac{4k+1}{2N}\pi\right)}{\xi^2 + \left(2\sin\left(\frac{2k}{N}\pi\right)\right)^2} \tag{4.27}$$

$$N\ \textit{odd:} \qquad C_N = \frac{4\sin\left(\frac{\pi}{2N}\right)}{\xi R_L}$$

$$N\ \textit{even:} \qquad L_N = \frac{4R_L\sin\left(\frac{\pi}{2N}\right)}{\xi} \tag{4.28}$$

The calculation must begin with Eqn. 4.26 if we have the value of R_S or in the reverse order, that is to say with Eqn. 4.28 if we have the value of R_L. Note that contrary to Butterworth filters, where the terminations can be identical, in the case of even-order Chebyshev filters, they must be different. The element values are tabulated in Tables 4-3 to 4-6 for different ripple values in the passband.

EXAMPLE: CALCULATION OF A THIRD-ORDER CHEBYSHEV PROTOTYPE FILTER

We would like to find the values of the reactive elements of a 3rd order Chebyshev filter having a ripple in the passband of less than 0.1 dB, a cutoff frequency of 1 rad/s and a source resistance $R_S = 1\Omega$.

From Eqn. 4.12 we get $\varepsilon = 0.1526204$. From Eqn. 4.25 we have $h = 2.36215$ and $\xi = 1.938812$. Knowing that $R_S = 1\Omega$, we can calculate C_1 from Eqn. 4.26:

$$C_1 = \frac{4\sin(\pi/6)}{\xi} = 1.03156F$$

$$C_1 L_2 = \frac{16\sin(\pi/6)\sin((3\pi)/6)}{\xi^2 + (2\sin(\pi/3))^2} = 1.183609$$

From which: $$L_2 = 1.1473966H$$

$$C_3 L_2 = \frac{16\sin((3\pi)/6)\sin((5\pi)/6)}{\xi^2 + (2\sin((2\pi)/3))^2} = 1.183609$$

From which: $$C_3 = 1.03156F$$

Note that according to Eqn. 4.28, R_L is equal to R_S.

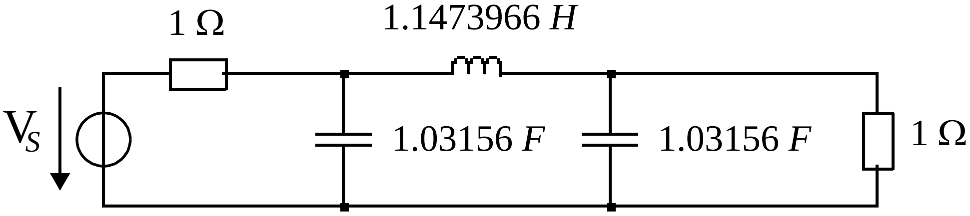


Fig 4-18: *Example of the calculation of a 3rd order Chebyshev prototype filter.*

SURFACE ACOUSTIC WAVE (SAW) FILTERS (1/2)

Surface Acoustic Waves (SAWs) are a special type of elastic wave that propagates along discontinuities such as the free surface of a solid (or the separation surface between two different elastic media). They were discovered theoretically by Lord Rayleigh, in 1885, during his studies of earthquakes. The amplitude of the mechanical deformations decreases exponentially inside the solid when getting farther from the surface such that the mechanical energy carried by the wave is confined, in a region about the thickness of the wavelength λ, under the surface (cf Fig. 4-19).

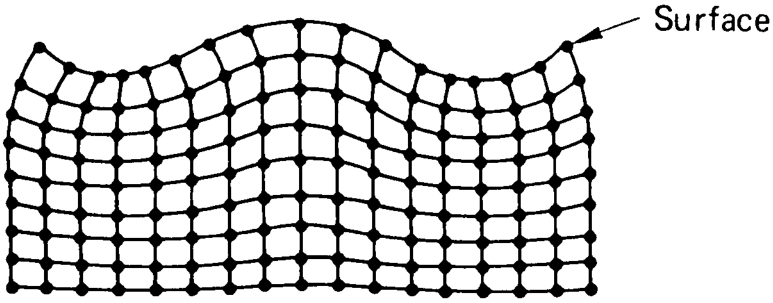


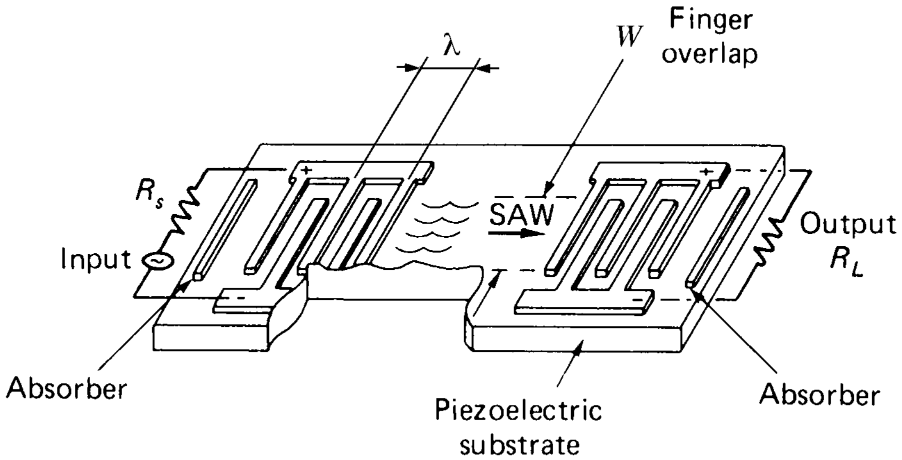
Fig 4-19: *Propagation of a surface wave.*

These waves, nondispersive, are characterized by slow propagation (average speed $v = 3\ km/s$) and a generally weak attenuation (on the order of 10^{-4} dB/λ, i.e. 0,01 dB/μs for lithium niobate YZ at 100 MHz).

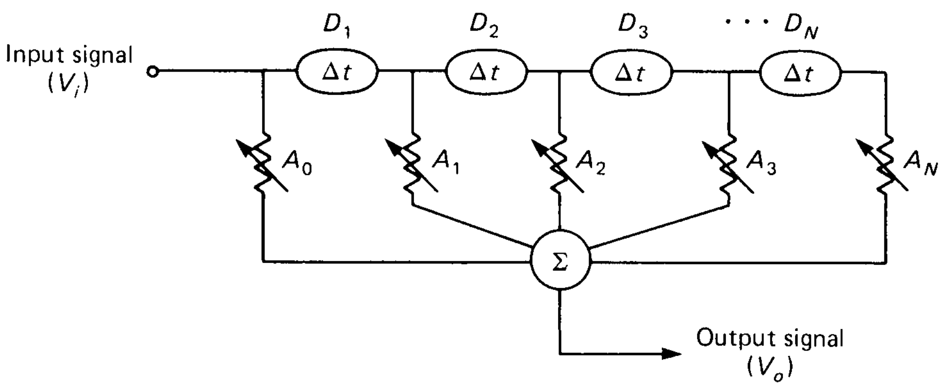
Since the signals to be filtered are usually electrical, the use of elastic phenomena requires transformations from mechanical energy to electrical energy and vice versa. Piezoelectric crystals in which there is a natural coupling between elastic and electrical phenomena are thus the material used. Indeed, if the substrate is piezoelectric, the deformations produced by the elastic wave induce local electric fields, which accompany the mechanical wave during its propagation. The electric field interacts with all the metal electrodes placed on the surface, which can also be connected to exterior circuits.

SURFACE ACOUSTIC WAVE (SAW) FILTERS (2/2)

The surface waves are generated and detected with transducers made of interdigitated metallic combs deposited on the substrate (cf Fig. 4-20 a)). The classic technology uses photolithography of a thin metallic layer, usually aluminum about 2000 Å thick, deposited on a polished monocrystal: one single mask level is usually enough and the fabrication yield is excellent.



a) Setup of a SAW filter.



b) Corresponding transversal filter.

Fig 4-20: *Diagram of a SAW filter.*

The surface acoustic wave filter corresponds to the transversal filter (FIR filter) whose diagram is shown in Fig. 4-20 b). The delay Δt is due to the spacing between each finger of the transducers, and the weighting factor A of the delayed signal is determined by the length W of each finger.

TRANSFER FUNCTION OF A SAW FILTER

For a regular comb, the elastic excitations due to different finger pairs add together to give a synchronous frequency $f_0 = v/\lambda$. If the frequency moves away from this value, the interference is no longer completely constructive and the resulting signal diminishes: the passband of a regular transducer is narrower when it has more fingers. If N is the total number of fingers, the frequency response is given by:

$$H(x) = \frac{\sin(x)}{x} \qquad with \quad x \equiv (N-1)\frac{\pi}{2}\frac{f-f_0}{f_0} \tag{4.29}$$

Other responses can be obtained either by changing the amplitude by appropriately weighting finger length (apodization) (cf Fig. 4-21 a)), or by changing the phase by weighting finger spacing (cf Fig. 4-21 b)).

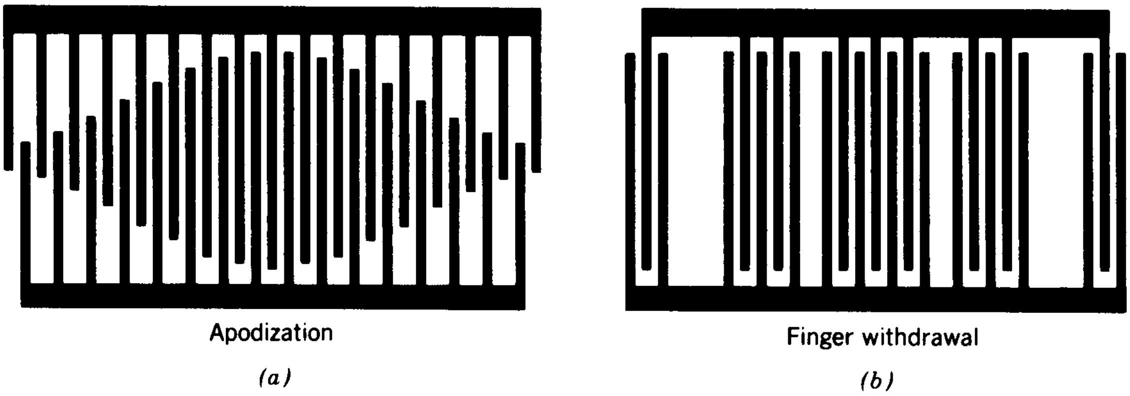


Fig 4-21: *Weighting the impulse response.*

The global transfer function is equal to the product of the relative functions of the emitter and receiver transducers. If one of the transducers has few fingers (meaning a wide passband), the transfer function is determined uniquely by the design of the other. In addition, if the second transducer with uniform aperture has few fingers, it will have high insertion loss. If the number of fingers increases, its frequency response, in sin x/x, will "round out" the global response of the filter. The use of two apodized transducers complicates the problem of synthesis.

TRANSDUCER EQUIVALENT CIRCUIT

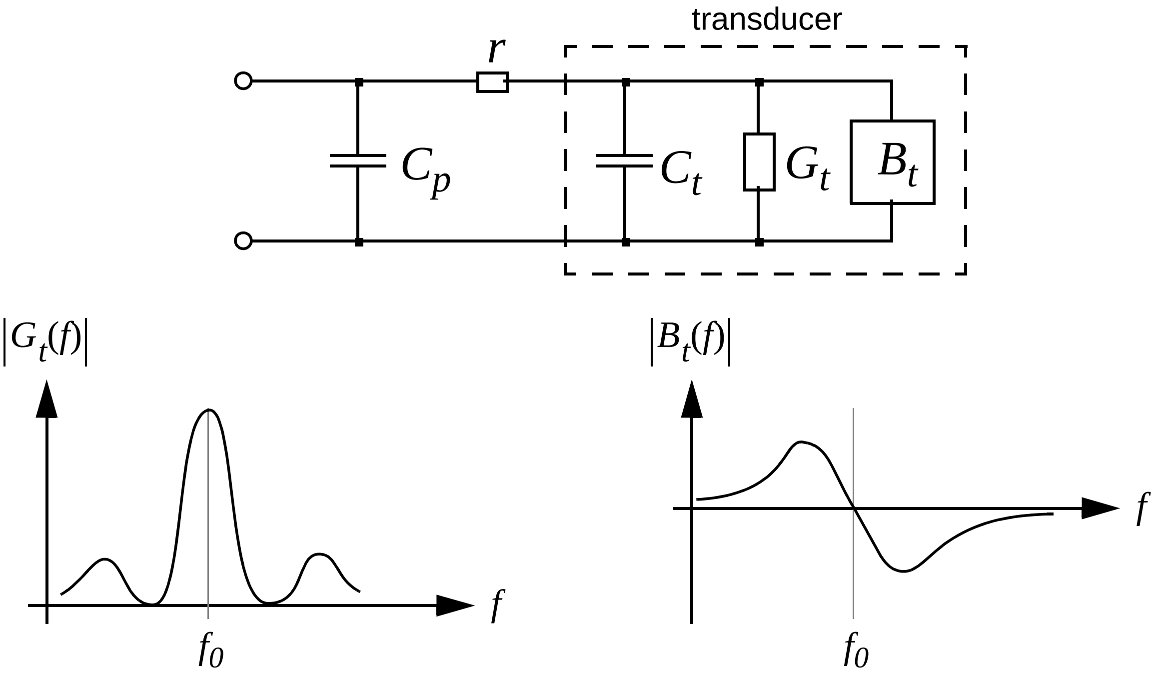


Fig 4-22: *Transducer equivalent circuit.*

The equivalent circuit of a transducer is shown in Fig. 4-22. The input admittance of the transducer (within the dotted lines of Fig. 4-22) is composed of three parallel elements given by:

$$C_t \equiv \frac{N-1}{2}\varepsilon W$$

$$G_t(f) \equiv G_{max}\left[\frac{\sin(x)}{x}\right]^2 \qquad (4.30)$$

$$B_t(f) \equiv G_{max}\frac{\sin(2x)-2x}{2x^2}$$

with:

$$G_{max} \equiv G_t(f_0) = 8k^2 f_0 C_t \frac{N-1}{2} \qquad (4.31)$$

where k^2 is the electromechanical coupling coefficient of the piezoelectric material, ε its permittivity and W the length of the transducer fingers. Two parasitic elements have been added: the resistance r of the electrodes and the capacitance C_p associated with the setup.

REFLECTIONS AND INSERTION LOSSES

Secondary effects change the ideal response of interdigitated transducers. The most important are related either to diffraction, the waveform spreading out if its opening is too small, or to reflections from the crystal edges and particularly from the other electrodes. Intratransducer reflections can be minimized by using double-finger electrodes, in which each finger is replaced by a pair of fingers of thickness $\lambda/8$ instead of $\lambda/4$ (cf Fig. 4-23).

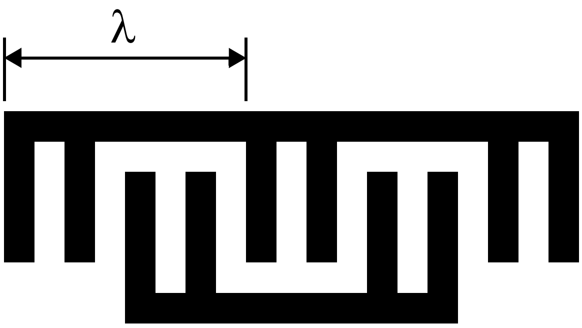


Fig 4-23: *Reduction of intratransducer reflections.*

The reflections from the two ports, due to the regeneration of elastic waves by the voltage created on the transducer electrodes, cause "triple transit" echoes that are as strong as the losses are weak. This effect can be reduced by slightly changing the device so that the insertion losses are equal or greater than 20 dB or by using double or unidirectional multiphased transducers at the price of a reduced bandwidth.

The insertion losses of surface acoustic wave filters stem mainly from the bidirectionality of the transducers and from the electrical mismatching, but also, especially at high frequency, from the parasitic resistance of the fingers and from the propagation losses in the substrate. In addition, by tolerating reasonable losses, one can only obtain, from a given material, a limited relative passband $\Delta f / f_0$, on the order of k, the square root of the electromechanical coupling coefficient.

LIMITS AND APPLICATIONS OF SAW FILTERS

Under 10 MHz, crystal dimensions lead to filters with modest performance. Above 1 GHz, major technological problems arise. But it is in the VHF and UHF ranges that SAW filters are the most advantageous. Limited by the dimensions of the crystals (several cm), the transition band of transversal filters remains higher than 100 kHz. It can be reduced by the use of resonators. It is difficult to give characteristic numbers, for insertion losses or for ripples in the band, because performance varies enormously both with the quality of the design and realization and also as a function of the filter design specifications. Thus a narrow-band filter can have just 2 or 3 dB of losses if the unidirectional structure is realized well, while a wide-band filter with small ripples in the band will have insertion losses of about 20 dB. A major advantage of SAW filters is that they don't require any frequency adjustment.

They are commonly used to realize the IF filters in TV receivers (cf Fig. 4-24).

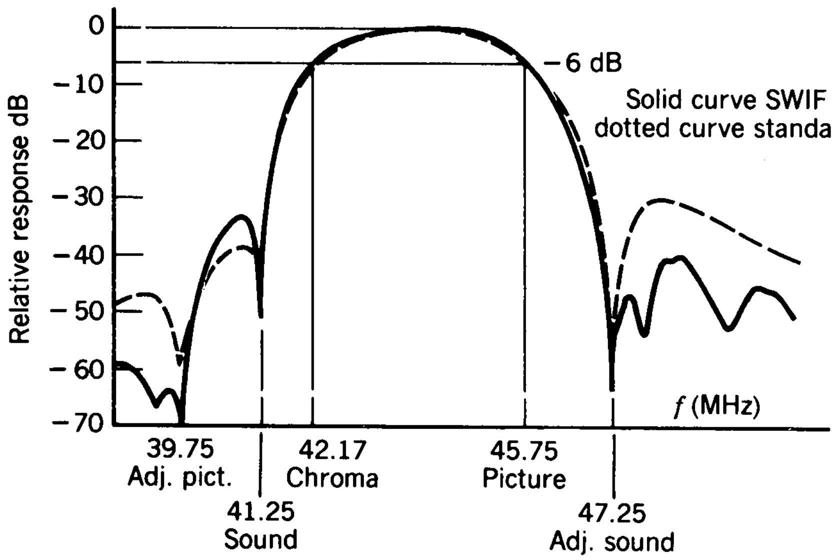


Fig 4-24: *Response of a SAW filter for IF TV.*

OTHER SAW FILTER STRUCTURES

The preceding description of surface wave filters implies an ideal model in which the waves propagate freely, without reflection. In practice, we seek to design devices in which these conditions are roughly satisfied. Nevertheless, there is another approach in which the deviation from the model of free waves is deliberately emphasized in order to take advantage of these effects.

The surface motion reflects from mechanical and electrical discontinuities. If we exclude the reflections from the crystal boundaries, difficult to control and usually avoided by using an absorber (cf Fig. 4-20), we mostly use reflections from arrays:

a) arrays of mechanical grooves: Regular grooves can be etched by ions on a crystal surface; an array made of ZnO or silica, or even metal can also be deposited. Thus, each line of discontinuity will be the source of a reflected wavelet. If the reflected wavelets from the different lines are in phase, the effect will be cumulative.

b) conductor arrays: Reflections from arrays of conductors are more subtle. The incident waves induce a current in the conductors, which then behave as transducers whose excitation is caused by the incident wave. The waves re-emitted in this way by the transducer are interpreted as "electrically" reflected (or diffused) waves. The amplitude of this diffusion depends strongly on the load: open circuit, short circuit, matched circuit, etc. One can even obtain a re-emission that is geometrically separated from the incident wave, which leads to multiple band couplers.

SAW RESONATOR FILTERS

By placing two transducers inside a surface wave cavity closed by two reflectors of one of the types decribed above, we obtain a resonator filter.

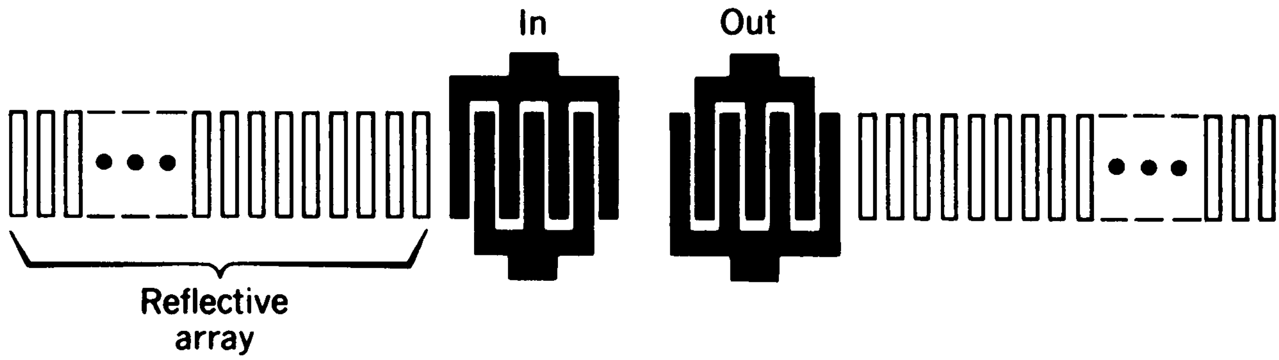


Fig 4-25: *Example of a SAW resonator filter.*

We can thus achieve quality factors on the order of 10'000. It is also possible to place the transducers outside the cavity, which makes their position less critical but increases insertion losses. We can also couple multiple cavities, for example by using multiple-band couplers, but this process is rarely used for more than two cavities.

MULTISTRIP ARRAY FILTERS

A multiple band coupler placed astride over an input track and an output track has the simple effect of shifting the phase of the piezoelectric waves. On the other hand, if the pitch of the array is modulated on both tracks, we get a selective reflection at certain frequencies, which allows the realization of electric diffusion filters. We can also use this procedure by reversing the propagation direction with reflection (MRA or multistrip reflective array) or on the contrary, by preserving the propagation direction with transmission (MTA or multistrip transmissive array).

REFLECTIVE ARRAY COMPRESSOR SAW FILTERS

Fig. 4-26 represents a double-reflection filter on two etched arrays, turned approximately 45° from the axis of propagation of the input and output waves. This process is very convenient for realizing dispersive filters (used in modern radar) in which the bandwidth x transmission time product reaches 10^4. A similar process which uses points instead of grooves has also been proposed.

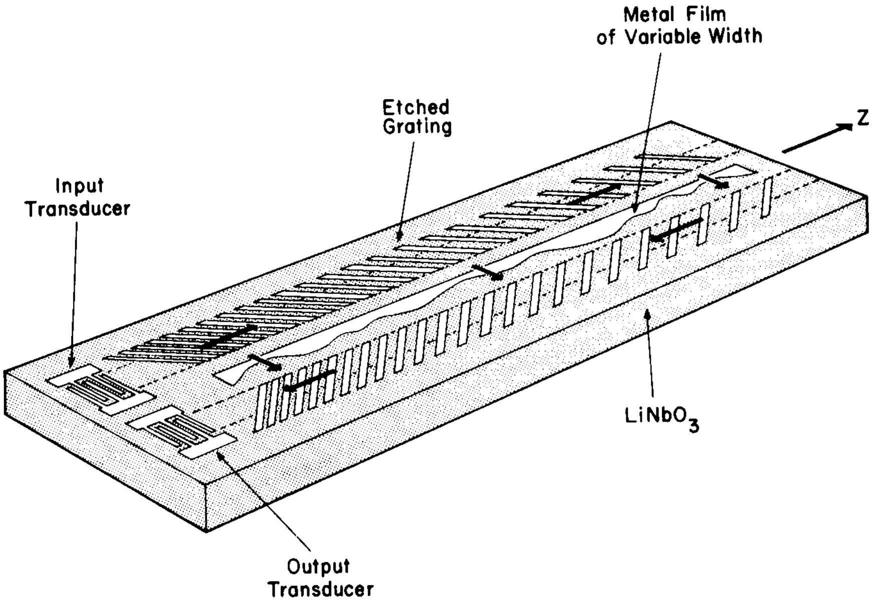
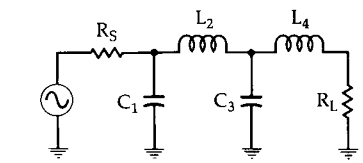


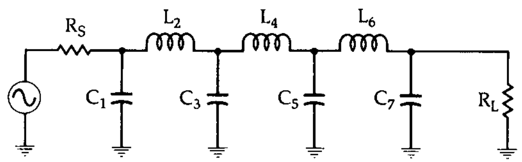
Fig 4-26: *Reflective array dispersive filter.*

ACTIVE SAW FILTERS

Finally, we would like to point out the possibility of looping back a line of surface waves on itself through an amplifier which almost completely compensates the losses. We thus obtain extremely narrow-band filters (overvoltage = several 10^4) that can be made tunable by inserting an electronic phase-shifter in the loop.

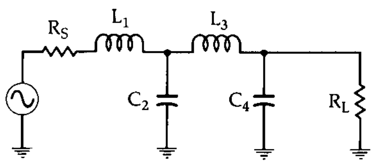
Table 4-2: *Butterworth prototype filter ($R_S \neq R_L$, $\varepsilon = 1$).*





n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.111	1.035	1.835		
	1.250	0.849	2.121		
	1.429	0.697	2.439		
	1.667	0.566	2.828		
	2.000	0.448	3.346		
	2.500	0.342	4.095		
	3.333	0.245	5.313		
	5.000	0.156	7.707		
	10.000	0.074	14.814		
	∞	1.414	0.707		
3	0.900	0.808	1.633	1.599	
	0.800	0.844	1.384	1.926	
	0.700	0.915	1.165	2.277	
	0.600	1.023	0.965	2.702	
	0.500	1.181	0.779	3.261	
	0.400	1.425	0.604	4.064	
	0.300	1.838	0.440	5.363	
	0.200	2.669	0.284	7.910	
	0.100	5.167	0.138	15.455	
	∞	1.500	1.333	0.500	
4	1.111	0.466	1.592	1.744	1.469
	1.250	0.388	1.695	1.511	1.811
	1.429	0.325	1.862	1.291	2.175
	1.667	0.269	2.103	1.082	2.613
	2.000	0.218	2.452	0.883	3.187
	2.500	0.169	2.986	0.691	4.009
	3.333	0.124	3.883	0.507	5.338
	5.000	0.080	5.684	0.331	7.940
	10.000	0.039	11.094	0.162	15.642
	∞	1.531	1.577	1.082	0.383
n	R_L/R_S	L_1	C_2	L_3	C_4

n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	0.900	0.442	1.027	1.910	1.756	1.389		
	0.800	0.470	0.866	2.061	1.544	1.738		
	0.700	0.517	0.731	2.285	1.333	2.108		
	0.600	0.586	0.609	2.600	1.126	2.552		
	0.500	0.686	0.496	3.051	0.924	3.133		
	0.400	0.838	0.388	3.736	0.727	3.965		
	0.300	1.094	0.285	4.884	0.537	5.307		
	0.200	1.608	0.186	7.185	0.352	7.935		
	0.100	3.512	0.091	14.095	0.173	15.710		
	∞	1.545	1.694	1.382	0.894	0.309		
6	1.111	0.289	1.040	1.322	2.054	1.744	1.335	
	1.250	0.245	1.116	1.126	2.239	1.550	1.688	
	1.429	0.207	1.236	0.957	2.499	1.346	2.062	
	1.667	0.173	1.407	0.801	2.858	1.143	2.509	
	2.000	0.141	1.653	0.654	3.369	0.942	3.094	
	2.500	0.111	2.028	0.514	4.141	0.745	3.931	
	3.333	0.082	2.656	0.379	5.433	0.552	5.280	
	5.000	0.054	3.917	0.248	8.020	0.363	7.922	
	10.000	0.026	7.705	0.122	15.786	0.179	15.738	
	∞	1.553	1.759	1.553	1.202	0.758	0.259	
7	0.900	0.299	0.711	1.404	1.489	2.125	1.727	1.296
	0.800	0.322	0.606	1.517	1.278	2.334	1.546	1.652
	0.700	0.357	0.515	1.688	1.091	2.618	1.350	2.028
	0.600	0.408	0.432	1.928	0.917	3.005	1.150	2.477
	0.500	0.480	0.354	2.273	0.751	3.553	0.951	3.064
	0.400	0.590	0.278	2.795	0.592	4.380	0.754	3.904
	0.300	0.775	0.206	3.671	0.437	5.761	0.560	5.258
	0.200	1.145	0.135	5.427	0.287	8.526	0.369	7.908
	0.100	2.257	0.067	10.700	0.142	16.822	0.182	15.748
	∞	1.558	1.799	1.659	1.397	1.055	0.656	0.223
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



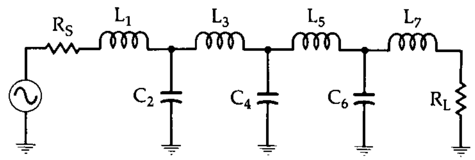
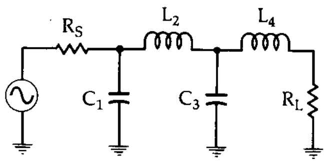
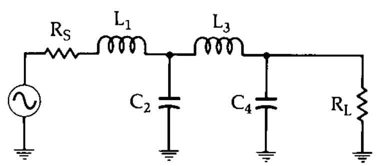
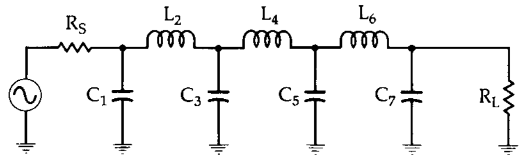


Table 4-3: *Chebyshev prototype filter ($A_p \leq 0.01$ dB).*



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.101	1.347	1.483		
	1.111	1.247	1.595		
	1.250	0.943	1.997		
	1.429	0.759	2.344		
	1.667	0.609	2.750		
	2.000	0.479	3.277		
	2.500	0.363	4.033		
	3.333	0.259	5.255		
	5.000	0.164	7.650		
	10.000	0.078	14.749		
	∞	1.412	0.742		
3	1.000	1.181	1.821	1.181	
	0.900	1.092	1.660	1.480	
	0.800	1.097	1.443	1.806	
	0.700	1.160	1.228	2.165	
	0.600	1.274	1.024	2.598	
	0.500	1.452	0.829	3.164	
	0.400	1.734	0.645	3.974	
	0.300	2.216	0.470	5.280	
	0.200	3.193	0.305	7.834	
	0.100	6.141	0.148	15.390	
	∞	1.501	1.433	0.591	
4	1.100	0.950	1.938	1.761	1.046
	1.111	0.854	1.946	1.744	1.165
	1.250	0.618	2.075	1.542	1.617
	1.429	0.495	2.279	1.334	2.008
	1.667	0.398	2.571	1.128	2.461
	2.000	0.316	2.994	0.926	3.045
	2.500	0.242	3.641	0.729	3.875
	3.333	0.174	4.727	0.538	5.209
	5.000	0.112	6.910	0.352	7.813
	10.000	0.054	13.469	0.173	15.510
	∞	1.529	1.694	1.312	0.523
n	R_L/R_S	L_1	C_2	L_3	C_4





n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	0.977	1.685	2.037	1.685	0.977		
	0.900	0.880	1.456	2.174	1.641	1.274		
	0.800	0.877	1.235	2.379	1.499	1.607		
	0.700	0.926	1.040	2.658	1.323	1.977		
	0.600	1.019	0.863	3.041	1.135	2.424		
	0.500	1.166	0.699	3.584	0.942	3.009		
	0.400	1.398	0.544	4.403	0.749	3.845		
	0.300	1.797	0.398	5.772	0.557	5.193		
	0.200	2.604	0.259	8.514	0.368	7.826		
	0.100	5.041	0.127	16.741	0.182	15.613		
	∞	1.547	1.795	1.645	1.237	0.488		
6	1.101	0.851	1.796	1.841	2.027	1.631	0.937	
	1.111	0.760	1.782	1.775	2.094	1.638	1.053	
	1.250	0.545	1.864	1.489	2.403	1.507	1.504	
	1.429	0.436	2.038	1.266	2.735	1.332	1.899	
	1.667	0.351	2.298	1.061	3.167	1.145	2.357	
	2.000	0.279	2.678	0.867	3.768	0.954	2.948	
	2.500	0.214	3.261	0.682	4.667	0.761	3.790	
	3.333	0.155	4.245	0.503	6.163	0.568	5.143	
	5.000	0.100	6.223	0.330	9.151	0.376	7.785	
	10.000	0.048	12.171	0.162	18.105	0.187	15.595	
	∞	1.551	1.847	1.790	1.598	1.190	0.469	
7	1.000	0.913	1.595	2.002	1.870	2.002	1.595	0.913
	0.900	0.816	1.362	2.089	1.722	2.202	1.581	1.206
	0.800	0.811	1.150	2.262	1.525	2.465	1.464	1.538
	0.700	0.857	0.967	2.516	1.323	2.802	1.307	1.910
	0.600	0.943	0.803	2.872	1.124	3.250	1.131	2.359
	0.500	1.080	0.650	3.382	0.928	3.875	0.947	2.948
	0.400	1.297	0.507	4.156	0.735	4.812	0.758	3.790
	0.300	1.669	0.372	5.454	0.546	6.370	0.568	5.148
	0.200	2.242	0.242	8.057	0.360	9.484	0.378	7.802
	0.100	4.701	0.119	15.872	0.178	18.818	0.188	15.652
	∞	1.559	1.867	1.866	1.765	1.563	1.161	0.456
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7

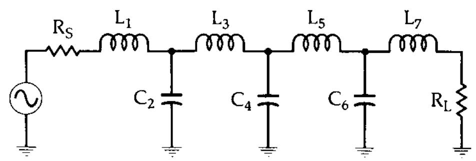
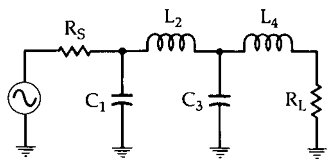
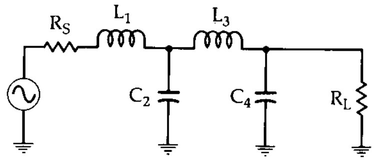
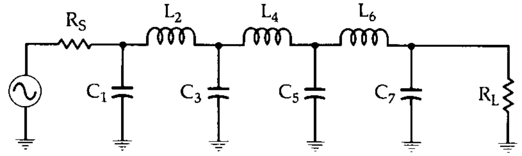


Table 4-4: *Chebyshev prototype filter ($A_p \leq 0.1$ dB).*



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.355	1.209	1.638		
	1.429	0.977	1.982		
	1.667	0.733	2.489		
	2.000	0.560	3.054		
	2.500	0.417	3.827		
	3.333	0.293	5.050		
	5.000	0.184	7.426		
	10.000	0.087	14.433		
	∞	1.391	0.819		
3	1.000	1.433	1.594	1.433	
	0.900	1.426	1.494	1.622	
	0.800	1.451	1.356	1.871	
	0.700	1.521	1.193	2.190	
	0.600	1.648	1.017	2.603	
	0.500	1.853	0.838	3.159	
	0.400	2.186	0.660	3.968	
	0.300	2.763	0.486	5.279	
	0.200	3.942	0.317	7.850	
	0.100	7.512	0.155	15.466	
	∞	1.513	1.510	0.716	
4	1.355	0.992	2.148	1.585	1.341
	1.429	0.779	2.348	1.429	1.700
	1.667	0.576	2.730	1.185	2.243
	2.000	0.440	3.227	0.967	2.856
	2.500	0.329	3.961	0.760	3.698
	3.333	0.233	5.178	0.560	5.030
	5.000	0.148	7.607	0.367	7.614
	10.000	0.070	14.887	0.180	15.230
	∞	1.511	1.768	1.455	0.673
n	R_L/R_S	L_1	C_2	L_3	C_4





n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.301	1.556	2.241	1.556	1.301		
	0.900	1.285	1.433	2.380	1.488	1.488		
	0.800	1.300	1.282	2.582	1.382	1.738		
	0.700	1.358	1.117	2.868	1.244	2.062		
	0.600	1.470	0.947	3.269	1.085	2.484		
	0.500	1.654	0.778	3.845	0.913	3.055		
	0.400	1.954	0.612	4.720	0.733	3.886		
	0.300	2.477	0.451	6.196	0.550	5.237		
	0.200	3.546	0.295	9.127	0.366	7.889		
	0.100	6.787	0.115	17.957	0.182	15.745		
	∞	1.561	1.807	1.766	1.417	0.651		
6	1.355	0.942	2.080	1.659	2.247	1.534	1.277	
	1.429	0.735	2.249	1.454	2.544	1.405	1.629	
	1.667	0.542	2.600	1.183	3.064	1.185	2.174	
	2.000	0.414	3.068	0.958	3.712	0.979	2.794	
	2.500	0.310	3.765	0.749	4.651	0.778	3.645	
	3.333	0.220	4.927	0.551	6.195	0.580	4.996	
	5.000	0.139	7.250	0.361	9.261	0.384	7.618	
	10.000	0.067	14.220	0.178	18.427	0.190	15.350	
	∞	1.534	1.884	1.831	1.749	1.394	0.638	
7	1.000	1.262	1.520	2.239	1.680	2.239	1.520	1.262
	0.900	1.242	1.395	2.361	1.578	2.397	1.459	1.447
	0.800	1.255	1.245	2.548	1.443	2.624	1.362	1.697
	0.700	1.310	1.083	2.819	1.283	2.942	1.233	2.021
	0.600	1.417	0.917	3.205	1.209	3.384	1.081	2.444
	0.500	1.595	0.753	3.764	0.928	4.015	0.914	3.018
	0.400	1.885	0.593	4.618	0.742	4.970	0.738	3.855
	0.300	2.392	0.437	6.054	0.556	6.569	0.557	5.217
	0.200	3.428	0.286	8.937	0.369	9.770	0.372	7.890
	0.100	6.570	0.141	17.603	0.184	19.376	0.186	15.813
	∞	1.575	1.858	1.921	1.827	1.734	1.379	0.631
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7

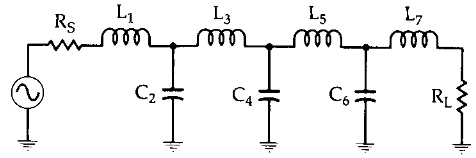
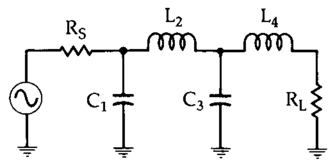
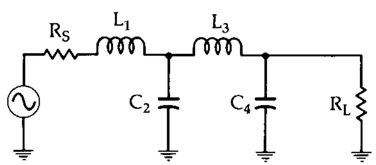
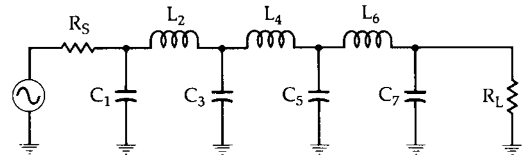


Table 4-5: *Chebyshev prototype filter ($A_p \leq 0.5$ dB).*



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.984	0.983	1.950		
	2.000	0.909	2.103		
	2.500	0.564	3.165		
	3.333	0.375	4.411		
	5.000	0.228	6.700		
	10.000	0.105	13.322		
	∞	1.307	0.975		
3	1.000	1.864	1.280	1.834	
	0.900	1.918	1.209	2.026	
	0.800	1.997	1.120	2.237	
	0.700	2.114	1.015	2.517	
	0.500	2.557	0.759	3.436	
	0.400	2.985	0.615	4.242	
	0.300	3.729	0.463	5.576	
	0.200	5.254	0.309	8.225	
	0.100	9.890	0.153	16.118	
	∞	1.572	1.518	0.932	
4	1.984	0.920	2.586	1.304	1.826
	2.000	0.845	2.720	1.238	1.985
	2.500	0.516	3.766	0.869	3.121
	3.333	0.344	5.120	0.621	4.480
	5.000	0.210	7.708	0.400	6.987
	10.000	0.098	15.352	0.194	14.262
	∞	1.436	1.889	1.521	0.913
n	R_L/R_S	L_1	C_2	L_3	C_4





n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.807	1.303	2.691	1.303	1.807		
	0.900	1.854	1.222	2.849	1.238	1.970		
	0.800	1.926	1.126	3.060	1.157	2.185		
	0.700	2.035	1.015	3.353	1.058	2.470		
	0.600	2.200	0.890	3.765	0.942	2.861		
	0.500	2.457	0.754	4.367	0.810	3.414		
	0.400	2.870	0.609	5.296	0.664	4.245		
	0.300	3.588	0.459	6.871	0.508	5.625		
	0.200	5.064	0.306	10.054	0.343	8.367		
	0.100	9.556	0.153	19.647	0.173	16.574		
	∞	1.630	1.740	1.922	1.514	0.903		
6	1.984	0.905	2.577	1.368	2.713	1.299	1.796	
	2.000	0.830	2.704	1.291	2.872	1.237	1.956	
	2.500	0.506	3.722	0.890	4.109	0.881	3.103	
	3.333	0.337	5.055	0.632	5.699	0.635	4.481	
	5.000	0.206	7.615	0.406	8.732	0.412	7.031	
	10.000	0.096	15.186	0.197	17.681	0.202	14.433	
7	1.000	1.790	1.296	2.718	1.385	2.718	1.296	1.790
	0.900	1.835	1.215	2.869	1.308	2.883	1.234	1.953
	0.800	1.905	1.118	3.076	1.215	3.107	1.155	2.168
	0.700	2.011	1.007	3.364	1.105	3.416	1.058	2.455
	0.600	2.174	0.882	3.772	0.979	3.852	0.944	2.848
	0.500	2.428	0.747	4.370	0.838	2.289	0.814	3.405
	0.400	2.835	0.604	5.295	0.685	5.470	0.669	4.243
	0.300	3.546	0.455	6.867	0.522	7.134	0.513	5.635
	0.200	5.007	0.303	10.049	0.352	10.496	0.348	8.404
	0.100	9.456	0.151	19.649	0.178	20.631	0.176	16.665
	∞	1.646	1.777	2.031	1.789	1.924	1.503	0.895
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7

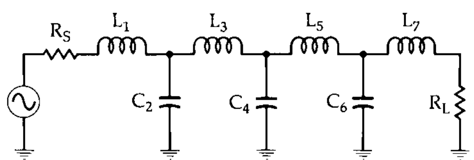
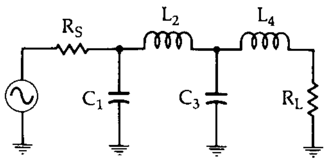
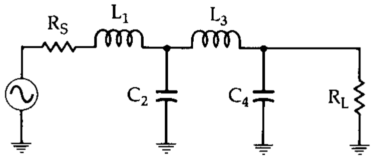
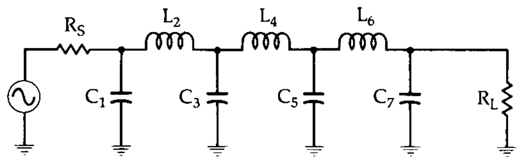


Table 4-6: *Chebyshev prototype filter ($A_p \leq 1$ dB).*



n	R_S/R_L	C_1	L_2	C_3	L_4
2	3.000	0.572	3.132		
	4.000	0.365	4.600		
	8.000	0.157	9.658		
	∞	1.213	1.109		
3	1.000	2.216	1.088	2.216	
	0.500	4.431	0.817	2.216	
	0.333	6.647	0.726	2.216	
	0.250	8.862	0.680	2.216	
	0.125	17.725	0.612	2.216	
	∞	1.652	1.460	1.108	
4	3.000	0.653	4.411	0.814	2.535
	4.000	0.452	7.083	0.612	2.848
	8.000	0.209	17.164	0.428	3.281
	∞	1.350	2.010	1.488	1.106
n	R_L/R_S	L_1	C_2	L_3	C_4





n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	2.207	1.128	3.103	1.128	2.207		
	0.500	4.414	0.565	4.653	1.128	2.207		
	0.333	6.622	0.376	6.205	1.128	2.207		
	0.250	8.829	0.282	7.756	1.128	2.207		
	0.125	17.657	0.141	13.961	1.128	2.207		
	∞	1.721	1.645	2.061	1.493	1.103		
6	3.000	0.679	3.873	0.771	4.711	0.969	2.406	
	4.000	0.481	5.644	0.476	7.351	0.849	2.582	
	8.000	0.227	12.310	0.198	16.740	0.726	2.800	
	∞	1.378	2.097	1.690	2.074	1.494	1.102	
7	1.000	2.204	1.131	3.147	1.194	3.147	1.131	2.204
	0.500	4.408	0.566	6.293	0.895	3.147	1.131	2.204
	0.333	6.612	0.377	9.441	0.796	3.147	1.131	2.204
	0.250	8.815	0.283	12.588	0.747	3.147	1.131	2.204
	0.125	17.631	0.141	25.175	0.671	3.147	1.131	2.204
	∞	1.741	1.677	2.155	1.703	2.079	1.494	1.102
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7